Lesson 4-4: Using Congruent Triangles (CPCTC)

Triangle congruence

We've now learned four ways of determining triangle congruence based on limited information:

1. SSS – all three corresponding sides are congruent
2. SAS – two sides and included angle are congruent
3. ASA – two angles and included side are congruent
4. AAS – two angles and non-included side are congruent

What we're going to work with today will likely seem completely obvious once you think about it.

Corresponding parts of congruent triangles (CPCTC)

Once you determine two triangles are congruent based on limited information, you can then turn around and conclude that all corresponding parts are congruent. We abbreviate this as CPCTC. Seems like a no-brainer, huh?

Using CPCTC

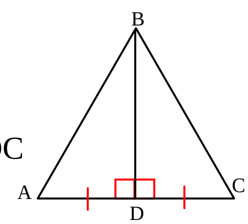
Often you will be given two triangles and asked to prove that specific parts are congruent. To do this, simply determine triangle congruence by SSS, SAS, ASA or AAS for the triangles in which the specific parts are corresponding. Here are some examples…

Examples

Explain how you can use SSS, SAS, ASA or AAS with CPCTC to prove each statement true.

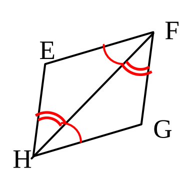
1. $\angle A \cong \angle C$

$\overline{AD} \cong \overline{DC}$	Given
$\angle ADB \cong \angle CDB$	All rt. $\angle$'s $\cong$
$\overline{BD} \cong \overline{BD}$	Reflexive POC
$\Delta ADB \cong \Delta CDB$	SAS
$\angle A \cong \angle C$	CPCTC



2. $\overline{HE} \cong \overline{FG}$

$\angle EFH \cong \angle GHF$	Given
$\overline{FH} \cong \overline{HF}$	Reflexive POC
$\angle EHF \cong \angle GFH$	Given
$\Delta EFH \cong \Delta GHF$	ASA
$\overline{HE} \cong \overline{FG}$	CPCTC



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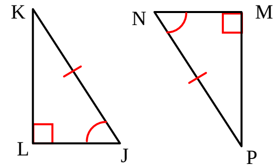
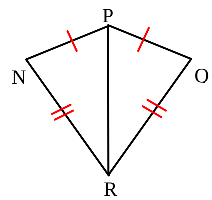
3. $\angle K \cong \angle P$

$\angle L \cong \angle M$	All rt. $\angle's \cong$
$\angle J \cong \angle N$	Given
$\overline{KJ} \cong \overline{PN}$	Given
$\Delta KLJ \cong \Delta PMN$	AAS
$\angle K \cong \angle P$	CPCTC



4. $\angle N \cong \angle Q$

$\overline{NP} \cong \overline{QP}$	Given
$\overline{NR} \cong \overline{QR}$	Given
$\overline{RP} \cong \overline{RP}$	Reflexive POC
$\Delta RNP \cong \Delta RQP$	SSS
$\angle N \cong \angle Q$	CPCTC



Systems of linear equations

Ah yes. Good old algebra. Here we are again. If you recall the marriage of algebra and geometry is called analytic geometry.

Do you recall solving two equations at once? This is called “solving a system of equations.” We need to review how to do this because in the next lesson we are going to be working with isosceles and equilateral triangles and will need to do this in order to find unknown angle measures.

Solving systems of linear equations

The simplest way of solving systems of linear equations is to use substitution to create a one-variable equation:

$16x - y = 2$
$3x - y = -11$

Solution:

$3x - y = -11$	Pick one of the two and equations.
$y = 3x + 11$	Solve it for *y*.
$16x - (3x + 11) = 2$	Subst. *3x + 11* for *y* in the other equation.
$13x - 11 = 2$	Solve for *x*.
$x = 1$	
$y = 3(1) + 11$	Subst. 1 for *x* and solve for *y*.
$y = 14$	
$14 = 16(1) - 2$	Subst. 1 for *x* & 14 for *y* in the other equation
$14 = 14$	to double-check your answer.

Our solution checks out in the true statement 14 = 14, we have one solution and the lines intersect at (1, 14).

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Systems of linear equations with no solution

If solving the equations results in a false statement, there is no solution for the equations. This means the lines never intersect.

$3x - y = 7$
$9x - 3y = 3$

Solution:

$9x - 3y = 3$	Pick the 2nd equation.
$y = 3x - 1$	Solve for *y*.
$3x - (3x - 1) = 7$	Subst. back into the 1st equation.
$1 = 7$	Solve for *x* – **FALSE!**

Since 1 = 7 is a false statement, this system of equations has no solution.

Systems of linear equations with infinite number of solutions

If solving the equations results in the true statement 0 = 0, the two equations represent the same line. You could think of the two lines lying on top of each other and hence intersect in an infinite number of points. Thus in this case there are an infinite number of solutions.

Assign homework

p. 204 #1-4, 7-13, 15, 19, 21, 28-36
p. 209 #1-9